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NAME

SAMPLE TEST, worth 100 points, Chapter 3

Show all work that leads to your answers. Good luck!

5 pts

Give a precise (ϵ–δ) definition of the mathematical sentence: $\lim_{x \to c} f(x) = l$

15 pts

All the following questions have to do with the true limit statement $\lim_{x \to 2} x^2 = 4$.

(2 pts) Very roughly, $\lim_{x \to 2} x^2 = 4$ says (fill in the blanks):

Whenever ___________ is close to ___________, it must be that ___________ is close to ___________.

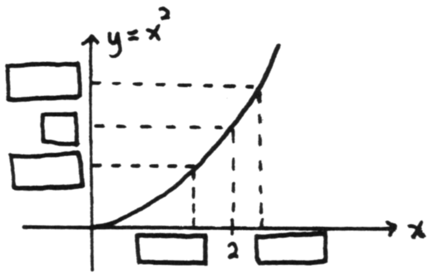
(2 pts) More precisely, the sentence says (fill in the blanks):

It is possible to get ___________ as close to ___________ as desired, merely by requiring that ___________ be sufficiently close to ___________.

The precise definition of $\lim_{x \to 2} x^2 = 4$ involves the sentence '$0 < |x - 2| < \delta$'.

(3 pts) For what value(s) of x is the sentence $0 < |x-2| < \delta$ true? Show these numbers on the number line below.

(8 pts) Fill in the boxes on the graph below with appropriate numbers/symbols that illustrate the '4-step process' showing that the limit statement $\lim_{x \to 2} x^2 = 4$ is true. Be sure to conclude with a 'δ that works'.



12 pts

TRUE or FALSE. (2 pts each) (Circle the correct response.)

T F For all real numbers a and b, $|a + b| \le |a| + |b|$.

T F If direct substitution into $\lim_{x \to c} f(x)$ yields a '$\frac{0}{0}$' situation, then the limit does not exist.

T F $(2 = 1)$ and $(1 + 1 = 2) \implies 4 = 3$

T F If an interval of real numbers is not open, then it is closed.

T F If f is continuous on $[a, b]$, then f must attain a maximum value on $[a, b]$.

T F If f is continuous on $[a, b]$, then there exists $c \in [a, b]$ for which $f(c) = \frac{f(a)+f(b)}{2}$.

8 pts

Evaluate the following limits, if they exist:

(3 pts) $\lim_{t \to -1} (t^3 - 2t^2 + 3)$

(5 pts) $\lim_{x \to 2} \dfrac{x^3 - x^2 - x - 2}{x^2 - 4}$

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4 pts

Prove that an implication $A \implies B$ is equivalent to its contrapositive. (HINT: Make a truth table! I've got you started.)

A	B	A ⟹ B	
T	T		
T	F		
F	T		
F	F		

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6 pts

(4 pts) The implication

$$\text{IF } x^2 = 4, \text{ THEN } x = 2$$

is false. Give a counterexample, by filling in the blanks:

Let $x =$ _________. Then _________________ is true, but _________________ is false.

(2 pts) TRUE or FALSE:

$$x = 2 \quad \implies \quad x^2 = 4$$

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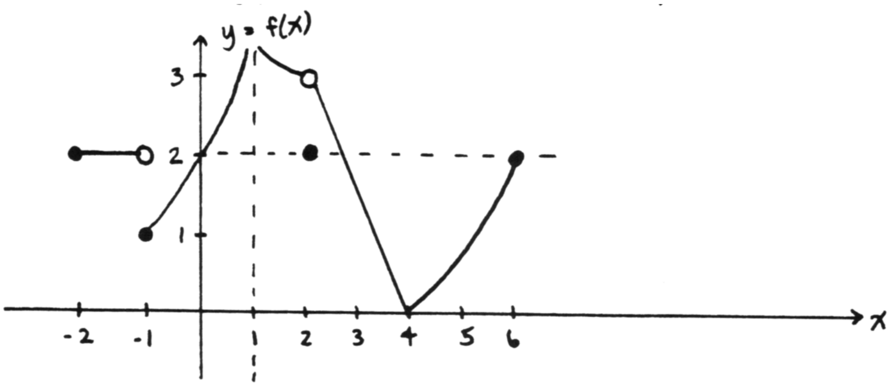
6 pts

Graph $f(x) = \frac{x^2-1}{x-1}$ in the space provided below.

24 pts

All the following questions refer to the graph of the function f shown below.



Find the following numbers/sets, if they exist. If they do not exist, so state. Be sure to write complete mathematical sentences. (2 pts each)

$f(-1)$ $\qquad \lim_{x\to -1} f(x)$ $\qquad \lim_{x\to -1.01} f(x)$

$\mathcal{D}(f)$ $\qquad \mathcal{R}(f)$ $\qquad \lim_{x\to -1^+} f(x)$

$\lim_{x\to 2} f(x)$ $\qquad \lim_{x\to -1^-} f(x)$ $\qquad f(-1.1)$

(2 pts) $\{x \mid f \text{ has a nonremovable discontinuity at } x\}$

(2 pts) $\{x \mid f(x) < 0\}$

(2 pts) Is f continuous on $[-1, 0]$? (YES or NO)

10 pts

Sketch the graph of a function f satisfying each set of requirements:

(5 pts) $\mathcal{D}(f) = (a, b)$, f is continuous on (a, b), $\lim_{x\to a^+} f(x) = 3$, $\lim_{t\to b^-} f(t) = -1$

(5 pts) f is continuous on $[0, 2]$, $f(0) = 1$, $f(2) = -1$. Must f attain a maximum value on $[0, 2]$? Why or why not?

10 pts

FILL IN THE BLANKS.

(5 pts) The Intermediate Value Theorem says: Let f be __________ on $[a, b]$. If D is any number between __________ and __________, then __________ a number d between __________ and __________ for which __________ .

(5 pts) A function f has a *removable discontinuity at* c whenever _______________ exists, but is not equal to _______________ .