

NAME (1 pt)

SAMPLE TEST, worth 100 points, Chapter 4

Show all work that leads to your answers. Good luck!

1. (14 pts) TRUE or FALSE. (Circle the correct response.)
- T F If f is continuous at x , then f is differentiable at x .
- T F $\mathbb{R} - (1, 2] = (-\infty, 1] \cup (2, \infty)$
- T F The Chain Rule tells us how to differentiate composite functions.
- T F Let K and n denote positive integers, and let $P(n)$ denote some statement about n . Suppose that $P(1)$ is true. Also suppose that if $P(K)$ is true, then $P(K + 1)$ must be true. Then $P(1007)$ is true.
- T F $\sum_{i=1}^3 i^{2i} = 1 + 2^4 + 3^6$
- T F $72 \cdot 71 \cdot \dots \cdot 49 = \frac{72!}{48!}$
- T F For all functions f and g , if f and g are differentiable at x , and $g'(x) \neq 0$, then $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)}{g'(x)}$.

2. (8 pts) Use the DEFINITION of derivative to find $f'(x)$ if $f(x) = x^2 - 1$. Be sure to write down complete mathematical sentences. I'll get you started:

$$f'(x) = \lim_{h \rightarrow 0}$$

3. (5 pts) Use Pascal's triangle to expand $(a + b)^4$.

4.
(28 pts)

Differentiate the following functions. Use any appropriate tools. Be sure to write complete and correct mathematical sentences.

(7 pts) $f(x) = \frac{\sqrt{2}}{\sqrt{x}}$

(7 pts) $y = xe^{2x-1}$

(7 pts) $g(t) = \frac{\ln t}{\sqrt[3]{t^2-1}}$

(7 pts) $y = (x+1)^{11}(e^x)(x^3)$ (A 'generalized product rule' may be helpful here.)

5.
(10 pts)

Sketch the graph of a function f satisfying each set of requirements:

(5 pts) f is continuous on $[0, 2]$, $f(0) = 1$, $f(2) = -1$, f is not differentiable at $x = 1$

(5 pts) $\mathcal{D}(f) = [1, 2]$, $f(1) = -1$, the average rate of change of f on $[1, 2]$ is 4, f is not linear on $[1, 2]$

6. (10 pts)
- (3 pts) Find the slope of the tangent line to the graph of $f(x) = x^3$ at $x = 1$.
- (4 pts) Find the EQUATION of the tangent line to the graph of $f(x) = x^3$ at $x = 1$.
- (3 pts) Find all points (x, y) on the graph of $f(x) = x^3$ where the tangent line has slope 12.

7. (4 pts)
- (2 pts) Give the PRIME notation for each of the following:
- the second derivative of f _____
 - the second derivative of f , evaluated at 2 _____
- (2 pts) Give the LEIBNITZ notation for each of the following. Assume that y is a function of x .
- the first derivative of y _____
 - the second derivative of y , evaluated at 0 _____

8. (5 pts)
- Suppose that f , g and h are differentiable everywhere. Then:

$$\frac{d}{dx} f(g(h(x))) = \underline{\hspace{10em}}$$

9. (8 pts)
- Give a precise statement of the Mean Value Theorem, and make a sketch that illustrates what this theorem is saying.

10. (7 pts)
- (Optional) Differentiate $f(x) = x^{2x}$.