## INDEX

## A

absolute value, 63
adding (a solution), 37
acceleration, 365
due to gravity, 367
additivity (of the definite integral), 412
and (the mathematical word), 25
truth table, 26
annotated, 36
antiderivatives:
definition, 344
on $[a, b], 409$
section on, 342-451
use for finding area, 405
antidifferentiation, 344
approaches, 109
approximating area by rectangles, 402
approximating (nearby function values), 221
approximating polynomials by highest order term, 313
arbitrary constant, 344
area:
between two curves, 428-435
of circle, 54
under a graph, 401
of triangle, 54
associativity (of the logical 'and'), 86
associative laws, 11
assumption in this text, 84
asymptotes:
section on, 330-338
average rate of change, 222
B
backwards (integrating 'backwards'), 412
between, 160
black boxes, 55
body (of proof), 225
bounded by, 430
bounds (getting bounds on function values), 268
brackets, 24
'building up' graphs from simpler pieces, 46, 77
C
calculus, 44
candidates for rational roots, 325
chain rule:
Leibniz notation for, 232
motivation for name, 232
precise statement, 231
section on, 228-237
change of variables formula, 425
circles, 264
classifying:
discontinuities, 148
an equation, 44
close (numbers being 'close'), 108
closed interval, 150
clubsuit symbol, iv
collection, 22
combinations (of functions), 82
common graphs, 45
complete and correct mathematical sentences, 1
complex numbers $(\mathbb{C}), 19$
complicated (products and quotients, differentiating), 261
composite functions:
associativity of function composition, 232
definition, 86
section on, 82-91
compound inequalities, 25
compounding (of interest), 102
concave (up and down), 299, 300
concise, 1
conditional sentences, 12
cone (finding volume using calculus), 439
conjecture, 44
conjure (What should the Leibniz notation conjure up?), 223
consequences
of the Intermediate Value Theorem, 281
of the Mean Value Theorem, 26
constant, 15
constant of integration, 345
contained in (subset), 83
continuity:
equivalent characterizations of, 224
idea, 108
on an interval, 149, 150
at a point (definition), 145
section on, 145-152
of sums, products, etc., 151
contradiction, 37, 134
logical justification for proof by contradiction, 135
contrapositive (of an implication), 176
conventions, 2
concerning definitions, 300
converse (of an implication), 290
conversely, 300
correspondence, (one-to-one), 92
counterexample, 168
counterpart antidifferentiation formula, 354
counting numbers, 22
critical points, 290
cube root, 213

## D

decimal expansions, 27
declarative, 2
decreasing functions, 276
decreasing info from the derivative, 278
definite integral, 408-417
definitions, 14
DeMorgan's Laws, 325
derivative:
of a constant, 205
definition, 193
of $e^{x}, 211$
of $e^{g(x)}, 236$
of $(g(x))^{n}$, General Power Rule, 233-234
of $\ln x, 212$
of $\ln g(x), 236$
of $x^{n}$, Simple Power Rule, 208
of sums and differences, 206
sliding constants out, 206
difference quotient, 184
differentiation, 193
direct proof (of an implication), 224
discontinuity:
definition, 146
nonremovable, 147
removable, 147
disk method, 436-443
distance:
between real numbers, 116, 428
between two points, 262
division, 35
do (facts can tell you what to do), 33
domain:
convention, 69
of a function, 69
of a sentence, 29
dominates (highest order term of a polynomial), 313
dummy variables:
in function notation, 61
of integration, 411
in limits, 109
in summation notation, 251
E
element, 22
empty set ( $($ ), 24
end-of-proof marker, 133
end-of-section exercises, iv
endpoints, 23
English usage (versus math. usage):
of the words 'open' and 'closed', 150
of the word 'or', 47
equality:
of functions, 156
of sets, 6
equivalence, 29-38
Esty, (Warren), 33
evaluate (a limit), 110
even functions, 318
even roots, 215
existence, 133
existence of antiderivatives, 425
explicit, 36, 257
exponential function, 99
exponents:
fractional, 216
in order of operations, 18
properties of, 216
expression, 1
extreme values and points, (local), 287
extreme (values, points), 171
F
fact, 33
factorable over the integers, 320
factoring quadratics, 320-322
factorial notation, 253
factoring, 80
falling object, 362-374
false sentences, 12
First Derivative Test, 295
First Derivative Test for endpoints, 295
for all (the mathematical phrase), 269-271
forms (indeterminate), 154
four-step process (for evaluating limits), 123
fractional exponent notation, 216
free-body diagram, 367
freshman's dream, 19
functions:
even and odd, 318-319
equality of, 156
precise view of, 217
section on, 54-67
fundamental theorem of algebra, 15
fundamental theorem of differential calculus, 266
fundamental theorem of integral calculus, 409
G
general power rule (differentiating $f(x)^{n}$ ), 234
global extrema, 288, 310
graphs:
common graphs, 45
section on, 39-53
gravity (acceleration due to), 367
greater than, 9
Greek letters, 7

## H

```
higher order derivatives:
Leibniz notation for, 255
prime notation for, 249
section on, 249-256
holes (solids with holes), 441
horizontal asymptotes, 331, 332
horizontal (lines), 48
horizontal line test, 56
hypothesis (plural, hypotheses):
of an implication, 167
of a theorem, 139
I
\(i\) (the imaginary number), 19
identity, 37
if \(A\), then \(B, 165\)
implication
contrapositive, 176
form and intuition, 165
hypothesis and conclusion, 167
proving, direct proof, 224
truth table, 166
implicit, 36, 257
implicit differentiation, 257-265
implied domain, 29
increasing functions, 276
increasing info from the derivative, 278
indefinite integrals, 344
indeterminate forms, 154-159
induction (proof by), 245
inequality:
graphing (in 2 variables), 44
symbols, 9
triangle, 139
infinitesimal slice, 428, 437
infinity \((\infty), 24\)
infinity (behavior at, limits involving), 330-339
infinitely differentiable, 249
inflection points, 299, 303
inputs, 54
in-section exercises, iv
inspection, 32
instantaneous (rates of change), 220-226
integers, 20, 27
integral sign, 344
integrals, 345
integrand, 344
integrating:
\(e^{x}, 359\)
\(\frac{1}{x}, 360,361\)
integration, 345
integration by parts formula, 391
```

with definite integrals, 426
intercept (slope-intercept form), 51
interchangeable, 29
interest:
compounding, 102
continuous compounding, 102
simple, 101
Intermediate Value Theorem, 160-169
intersection points, 433
intersection (of sets), 83
interval notation, 23
intuition (for developing formulas using the definite integral), 418
inverse functions:
finding $f^{-1}, 96$
relationship between $f$ and $f^{-1}, 95$
relationship between graphs of $f$ and $f^{-1}, 97$
section on, 94-98
irrational numbers, 20
J
jigsaw puzzles, 434
K
keywords, v
L
language, iii, 1
Leibniz notation:
for the chain rule, 232
for the derivative, 204
for higher order derivatives, 255
less than, 9
limits:
definition, 121
the idea, 108-119
left-hand, 130
operations with, 138
of polynomials, 142
properties of, 136
right-hand, 130
uniqueness, 134
limits of integration (upper and lower, for integration), 408
linear equations, 44
linearity of differentiation, 347
linearity of integration, 348, 411
lines:
example, 44
horizontal, 48
non-vertical, non-horizontal, 49
point-slope form, 190
slope-intercept form, 51
standard form, 51
vertical, 48
list, 22
local extrema, 287
locally (a function), 258
logarithm:
the natural logarithm function, 100
properties of, 218
logarithmic differentiation, 262
losing (a solution), 37
M
magnitude, 140
mapping diagrams, 60
MATLAB, iii
maximum (global), 310
maximum (local), 287
maximum (value), 171
Max-Min Theorem:
use in finding area under a graph, 402
precise statement, 174
section on, 171-177
mean (average), 266
Mean Value Theorem:
consequences of, 268
precise statement, 266
section on, 266-271
mentally solving an equation, 13
minimum (global), 310
minimum (local), 287
minimum (value), 171
minus, 8
motion along a line, 362-364
N
$n^{\text {th }}$ root, 213
$n$-tuple, 17
negating ' $A$ and $B$ ', 325
negating ' $A$ or $B$ ', 325
negative, 8
Newton's Second Law, 367, 368
nondecreasing functions, 277
nonincreasing functions, 277
nonnegative, 27
nonpositive, 27
norm:
mathematical tool for measuring size, 419
of a partition, 419
notation (for the definite integral), 418, 421-422
noun, 1
number line approach, 282
O
oblique asymptotes, 334
odd functions, 318
odd roots, 214
one-to-one functions:
precise view of, 217
section on, 92-94
one-sided limits, 130
open intervals, 150
operator $\left(\frac{d}{d x}\right.$ operator), 204
or (the mathematical word, truth table), 47
order (higher order derivatives), 249
ordered pair, 41
origin, 40
outputs, 54
P
parentheses, 24
particular solutions, 358, 359, 381
partition (of an interval), 418
parts formula (integration by parts), 391
Pascal's triangle, 210
pattern, 34
$\pi, 20$
place holders, 13
point, 41
point-slope form (for lines), 190
polynomial, 80
positive, 8
powerful, 1
precise, 1
predicting (nearby function values), 221
prime (notation for the derivative), 204
product of real numbers:
When is it negative?, 280
When is it positive?, 280
product rule for differentiation:
generalizing to more than 2 factors, 243
precise statement, 239
products (differentiating), 239
proving:
an implication, 224
punctured (graphs), 72
Q
quadrant, 40
quadratic formula, 24
for factoring quadratic polynomials, 322
quick quiz, v
quotients (differentiating), 239
quotient function, 85
quotient of integers, $(\mathbb{Q}), 27$
quotient rule for differentiation, 244
R
radicals, 213
range (of a function), 88
rates of change:
average rate of change, 222
idea, 220
instantaneous, section on, 220-226
rationalizing, 158
rational numbers $(\mathbb{Q}), 20,27$
Rational Root Theorem, 324
real numbers, 8
reconstructing a function from its derivative, 202
rectangular coordinate system, 39-40
relationship between differentiability and continuity, 223
Remainder Theorem, 328
removable discontinuity, 147
restricted equal sign, 155
revolution (solid of), 436
Riemann sums, 418, 420
right-hand limit, 130
root (of a polynomial), 80
roster, 22
S
secant line, 184
second derivative:
section on, 299-307
Second Derivative Test, 306
sentences, 2
set-builder notation, 23
sets:
intersection, 83
section on, 22-28
subset, 83
subtraction, 195
union, 64
shape (determined by $f^{\prime}$ ), 343
shell method, 444-449
shifting graphs left and right, 76
Simple Power Rule (differentiating $x^{n}$ ), 208
Simple Power Rule (integrating $x^{n}$ ), 355
singleton ('treat it as a singleton' technique), 142
slope:
definition, 49
formula for, 49
'no slope' versus 'zero slope', 199
slope-intercept form, 51
smooth (functions), 249
solution, 16
solution set, 24
solving an equation, 13
speed, 365
sphere, (finding volume using calculus), 438, 447
square root, 65-66, 213
standard form (of a line), 51
star symbol, iv
strength of operations, 18
stronger, (differentiability is 'stronger than' continuity), 224
subset, 83
substitution (direct substitution for limits), 136
substitution technique for integration, 376-384
substitution technique applied to definite integrals, 423-427
subtraction:
set subtraction, 195
a special kind of addition, 34
sum function, 85
summation notation, 251
summation (definite integral, infinite summation process), 422
symmetry:
about line $y=x, 97$
about the origin, 318
about $y$-axis, 318
synthetic division, 327-328
systematic approach to graphing, 309

## T

tangent lines, 182-190
$\mathrm{T}_{\mathrm{E}} \mathrm{X}$, v
text style:
for limits, 109
for sums, 251
theorem, 15
topological, 174
transforming problems, 376
translating mathematical sentences, 33
triangle inequality, 139
triangle, Pascal's, 210
truth of sentences, 4,12
truth table:
definition, 26
for 'and', 26
for $\Longleftrightarrow, 271$
for $\Longrightarrow, 166$
for 'or', 47
$n$-tuple, 17
two-column format, iii
U
unambiguous, 11
undefined, $\left(0^{0}\right), 19$
undoing differentiation, 342
union (set), 64
unique (related to functions), 54
uniqueness (a typical uniqueness argument), 133
universal set:
definition, 13
sometimes omitted in 'for all' sentences, 270
V
values (versus points), 310
variable:
dummy, 61
section on, 12-17
vectors, 366
velocity, 363,365 vertical asymptotes, 330
vertical lines, 48
vertical line test, 56
volume of a solid of revolution:
using disks, 436-443
using shells, 444-448
volume of a sphere, 54
W
well-defined, 22
X
$x$-axis, 40
Y
$y$-axis, 40
Z
zahlen, 27
zero factor law, 46-48
zero of a polynomial, 80

