# SATstuff#5

This week we'll review problems involving algebra.

**INTRODUCTION TO ALGEBRA**

Algebra gets its power from using *variables*, which are just letters used to 'hold' numbers. Indeed, a good way to think of a variable (like $x$) is a *container*, or a drawer in a filing cabinet.

If the drawer is labeled "$x$, real numbers" then you can reach in and pick out any real number (like $2$ or $1.3$ or $\sqrt{5}$ or $\pi$).

If the drawer is labeled "$n$, integers" then you can reach in and pick out any integer—the numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

QUICK QUESTIONS:

(1) How many variable(s) are in the expression $x^2 + 2x + 1$?

Answer: Only one; the variable $x$.

As long as only a single letter (like $x$) is used, there's only one variable.

The letter can appear any number of times.

(2) Evaluate the expression $x^2 + 2x + 1$ when $x = -3$.

'Evaluate' means to plug the specified number(s) in.

Usually, you also want to 'simplify' the result, which means to get a simpler name.

Answer: $x^2 + 2x + 1 = \overbrace{(-3)^2 + 2(-3) + 1}^{\text{evaluate}} = \overbrace{9 - 6 + 1 = 4}^{\text{simplify}}$

**EXPRESSIONS—the 'nouns' of math**

Math has EXPRESSIONS, which are like English nouns.

These are just names given to things we want to talk about in math, like $9$, $x^2 - y^2$, or $\frac{12}{3}$.

The most common thing you do with expressions is to RENAME THEM, because different names are better for doing different things!

- $9$ can be re-named as $3^2$ (showing that it is a perfect square)
- $x^2 - y^2$ can be re-named as $(x - y)(x + y)$ (FOIL it out! The Outers and Inners cancel.)
- $\frac{12}{3}$ can be re-named as $4$

QUICK QUESTIONS:

(3) Rename $\frac{3}{5}$ with a denominator of $10$.

Answer: $\frac{3}{5} \cdot \frac{2}{2} = \frac{6}{10}$

Multiplying by $1$ is a favorite way to rename:

it doesn't change where the number lives on a number line, but it changes its name!

The name $\frac{6}{10}$ is much better if we want the decimal name, $0.6$.

(4) Write $x^2 + 5x + 6$ as a product.

Answer: Writing something as a product (where the last operation is multiplication) is called FACTORING.

Here, we want two numbers that multiply to $6$ and add to $5$; the numbers $2$ and $3$ work.

Thus, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

These two names are the same for ALL values of $x$ (try a few on your calculator).

You spend lots of time in algebra courses factoring things, because the resulting name is MUCH MORE USEFUL (for reasons to be seen).

## MATHEMATICAL SENTENCES

Math also has SENTENCES:

- a sentence must express a complete thought
- it makes sense to ask if a sentence is (always) true, or (always) false, or sometimes true/sometimes false
- sentences have verbs

For example, $x - 3 = 0$ is a mathematical 'sentence':

- the verb is '='
- the only number that makes it true is 3; all other numbers make it false

A mathematical sentence that uses the verb '=' is called an EQUATION.
A mathematical sentence that uses one of these verbs is called an INEQUALITY:

$<$ less than
$>$ greater than
$\leq$ less than or equal to
$\geq$ greater than or equal to

Sentences ALSO have lots of different 'names'!!
We 'transform' sentences to make them easier to work with.
For example, the equations $2x - 3 = 5$ and $x = 4$ certainly LOOK different, but they are 'the same' in a VERY important way—they always have the same truth values!
Any value of $x$ that makes $2x - 3 = 5$ TRUE also makes $x = 4$ TRUE.
Any value of $x$ that makes $2x - 3 = 5$ FALSE also makes $x = 4$ FALSE.
So, these two equations always have the same TRUTH VALUES, and are said to be EQUIVALENT.

## TRANSFORMING TOOLS FOR SENTENCES

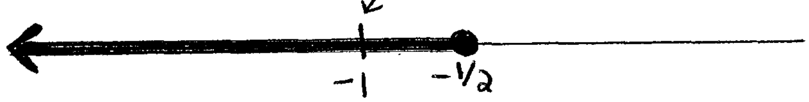
You can add/subtract the same number to/from both sides of any equation/inequality, and it won't change its truth value.
You can multiply/divide both sides of any equation by any NONZERO number, and it won't change its truth value.
If you multiply/divide both sides of an inequality by a NEGATIVE number, then you must also change the direction of the inequality symbol.
To 'SOLVE AN EQUATION' (or inequality) means to find all the number(s) that make it true.
Just write a nice clean list of equivalent equations:

$2x - 3 = 5$ (original equation)
$2x = 8$ (add 3 to both sides)
$x = 4$ (divide both sides by 2)

When is $x = 4$ true?
Obviously, when $x$ is 4, and that's the only number that makes it true.
So, when is $2x - 3 = 5$ true? Only when $x$ is 4.

QUICK QUESTION:
(5) Solve the inequality: $5 - 2x \geq 6$
Answer:
$5 - 2x \geq 6$ (original inequality)
$-2x \geq 1$ (subtract 5 from both sides)
$x \leq -\frac{1}{2}$ (divide both sides by $-2$; change the direction of the inequality symbol)

"SPOT-CHECK":
$5 - 2(-1) \geq 6$?
$7 \geq 6$ YES!!



Here's a typical SAT problem:
If $3x + 5 = 4$, then what is $3x - 1$?
You try it, before looking at the solutions below:

SOLUTION #1
(Solve for $x$, then substitute into desired expression)
$3x + 5 = 4$
$3x = -1$
$x = -\frac{1}{3}$
So:
$3x - 1 = 3(-\frac{1}{3}) - 1 = -1 - 1 = -2$

SOLUTION #2
(Write an equivalent equation that TELLS you what you want to know!)
You're interested in the expression $3x - 1$, so do something to both sides of the equation that brings this into the picture:
$3x + 5 = 4$
$(3x + 5) - 6 = 4 - 6$ (subtract 6 from both sides)
$3x - 1 = -2$
Quicker, easier, and no working with fractions!

YOU TRY THESE:

(6) If $2x + 1 = 7$, then what is $2x + 9$?

(7) If $2x + 1 = 7$, then what is $6x + 3$?

(8) If $2x + 1 = 7$, then what is an expression involving $x$ that will equal $5$?

(6) $2x + 9 = 15$ (7) $6x + 3 = 21$ (8) $2x - 1 = 5$

PRACTICE PROBLEMS:
(These incorporate ideas from earlier SATStuff sections!)

1. If $y = \frac{x^2 + x}{x}$ and $x = 13$, then $y =$

   (A) 14 (B) 26 (C) 78 (D) 169 (E) 172

2. Let a "$k$-triple" be defined as $(\frac{k}{2}, k, \frac{3}{2}k)$ for some number $k$.
   Which of the following is a $k$-triple?

   (A) $(0, 5, 10)$ (B) $(4\frac{1}{2}, 5, 6\frac{1}{2})$ (C) $(25, 50, 75)$ (D) $(250, 500, 1000)$ (E) $(450, 500, 650)$

3. If $a$, $b$, $c$, and $d$ are consecutive even integers and $a < b < c < d$, then $d - a =$

   (A) $-8$ (B) $-6$ (C) 3 (D) 6 (E) 8

4. The average (arithmetic mean) of the first three numbers in a certain series is 10. If the average of the first 2 numbers is 10, then what is the third number?

   (A) 0 (B) 5 (C) 10 (D) 20 (E) cannot be determined from the information given

5. If $x$ is the average (arithmetic mean) of $a$ and $b$, then in terms of $x$ and $c$, what is the average of $a$, $b$, and $c$?

   (A) $x + c$ (B) $\frac{x+c}{2}$ (C) $\frac{x+c}{3}$ (D) $\frac{2x+c}{2}$ (E) $\frac{2x+c}{3}$

6. If $p = 0.5q$ and $q = 0.5r$, then what is the value of $p$ in terms of $r$?

   (A) $2.5r$ (B) $2.0r$ (C) $1.0r$ (D) $0.25r$ (E) $0.1r$

7. If $x = yz$, which of the following must be equal to $xy$?

   (A) $yz$ (B) $yz^2$ (C) $y^2z$ (D) $\frac{x}{y}$ (E) $\frac{z}{x}$

8. The average (arithmetic mean) of $a$, $b$, $s$, and $t$ is 6 and the average of $s$ and $t$ is 3. What is the average of $a$ and $b$?

   (A) 3 (B) $\frac{9}{2}$ (C) 6 (D) 9 (E) 12

(This week, we should have time to go over problems from previous SAT sessions.)

1 A 2 C 3 D 4 C 5 E 6 D 7 C 8 D