

## SAT stuff #7

About one-third of the math problems on the SAT involve Geometry—mixed in with other ideas!

You are GIVEN many of the math formulas that you need at the beginning of each math portion:

### COMMON ANGLES

One complete revolution is 360 degrees (denoted  $360^\circ$ ).

So,  $1^\circ$  is  $\frac{1}{360}$  of a complete revolution.

A  $90^\circ$  angle is called a *right angle* and is always marked with a little box (see below).

Make sure you know what common angles look like:

### YOU TRY THESE:

(Assume all angles are measured in degrees.)

(a) What is  $x + y$ ?

(b) What is  $2(x + y)$ ?

(c) What is  $\frac{x + y}{3}$ ?

**INTERSECTING LINES**

Whenever two lines intersect, the angles opposite each other always have the same measure:

**PARALLEL LINES**

Lines that have the same "slant" (that is, the same slope) are called PARALLEL.

Whenever two parallel lines are cut by another (non-perpendicular) line, there are only TWO different angles that emerge!

All the bigger angles are the same, and all the smaller angles are the same.

BIGGER + SMALLER always equals  $180^\circ$ .

**PERPENDICULAR LINES**

Lines that meet at a  $90^\circ$  angle are called *perpendicular*.

If two lines are perpendicular, then their slopes are opposite reciprocals (change the sign, and flip).

For example, lines with slopes of 3 and  $-\frac{1}{3}$  are perpendicular.

Also, lines with slopes of  $-\frac{2}{3}$  and  $\frac{3}{2}$  are perpendicular.

YOU TRY THESE:

## TRIANGLES

The sum of the interior angles in ANY triangle is  $180^\circ$ .

If angles are the same, then their opposite sides are the same (and vice versa).

An *equilateral* triangle has all sides the same length (so all angles are the same, too... so each angle is  $60^\circ$ .)

An *isosceles* triangle has two sides the same length (so the opposite angles are the same, too).

Indeed, in any triangle:

- the longest side is opposite the biggest angle
- the shortest side is opposite the smallest angle
- the medium side is opposite the medium angle

Every triangle has three different base/height pairs, as shown below:

The area of ANY triangle is  $\frac{1}{2}(\text{base})(\text{height})$ .

YOU TRY THESE:

## **RIGHT TRIANGLES**

An *right* triangle has a  $90^\circ$  angle; the remaining two angles must add to  $90^\circ$ .

The longest side (the one opposite the  $90^\circ$  angle) is called the **HYPOTENUSE**.

The two shorter sides are called the **LEGS**.

The **PYTHAGOREAN THEOREM** gives a beautiful relationship between the lengths of the sides in any right triangle:

If  $a$  and  $b$  are the two shorter sides, and  $c$  is the hypotenuse, then:

$$a^2 + b^2 = c^2$$

Also, if the lengths of a triangle satisfy this equation (i.e., make it **TRUE**), then the triangle **MUST BE** a right triangle!

**YOU TRY THESE:**

## SPECIAL TRIANGLES

Here are a couple right triangles with particularly simple sides:

notice that  $3^2 + 4^2 = 5^2$  and  $5^2 + 12^2 = 13^2$ .

(Numbers like  $3 - 4 - 5$  and  $5 - 12 - 13$  are called *Pythagorean triples*.)

When you *scale* a triangle (that is, multiply ALL its sides by the same number), then the angles don't change.

Triangles with the same angles are called SIMILAR triangles.

So,  $6 - 8 - 10$  and  $9 - 12 - 15$  and  $10 - 24 - 26$  are all Pythagorean triples. (Check them out!)

Be on the lookout for triangles that correspond to Pythagorean triples!

### YOU TRY THESE:

(a) Suppose you're given a right triangle.

The lengths of one leg is 4 and the hypotenuse has length 5.

What is the length of the remaining side?

(b) Suppose you're given a right triangle.

The lengths of its legs are 3 and 4.

What is the length of the hypotenuse?

(c) Suppose you're given a triangle.

The lengths of its sides are 6, 8, and 10.

Is it a right triangle? If so, which side is opposite the right angle?

## MORE SPECIAL TRIANGLES

In the formula sheet, you're given two more special triangles: the  $30^\circ - 60^\circ - 90^\circ$  triangle, and the  $45^\circ - 45^\circ - 90^\circ$  triangle.

They look like this:

Here's how to use this information:

- In a  $30^\circ - 60^\circ - 90^\circ$  triangle, the shortest side is opposite the  $30^\circ$  angle.  
The hypotenuse is **TWICE** as long as the shortest side.  
The medium-length side is  $\sqrt{3}$  times as long as the shortest side ( $\sqrt{3} \approx 1.7$ ).
- In a  $45^\circ - 45^\circ - 90^\circ$  triangle, the hypotenuse is  $\sqrt{2}$  times as long as each leg ( $\sqrt{2} \approx 1.4$ ).

YOU TRY THESE:

## CIRCLES

Give me a POINT (call it  $P$ ) and a POSITIVE NUMBER (call it  $r$ ),  
and I can make you a circle:

it's ALL the points that are distance  $r$  from  $P$ .

The point  $P$  is called the *center* of the circle,  
and the number  $r$  is called the *radius* of the circle.

The *diameter* of the circle is twice the radius.

The CIRCUMFERENCE,  $C$ , of a circle is the distance around:  $C = 2\pi r$

Or, in terms of the diameter ( $d = 2r$ ):  $C = \pi d$

Here's a good way to think of this:

the distance AROUND any circle is a little more than three times the distance straight through the center to the opposite side!

The AREA of a circle,  $A$ , is found by squaring the radius, and multiplying by  $\pi$ :  $A = \pi r^2$

Since  $\pi$  is a little more than 3, here's a good way to think of this:

the area of the circle is a little more than 3 times the area of the square that is made from the radius.

(By the way, SAT gives you the circumference and area formulas for a circle.)

A *tangent* line to a circle is a line that is perpendicular to a radius; it touches the circle (as shown below) at only one point.

So

YOU TRY THESE: