SAMPLE PREREQUISITE PROBLEMS: ALGEBRA II

(no calculators allowed)

Multiplication Tables (through 12)

(You will have two minutes to do the following 24 multiplication problems.)

$2 \times 6 =$	$3 \times 2 =$	$4 \times 9 =$	$5 \times 2 =$
$8 \times 8 =$	$9 \times 3 =$	$10 \times 7 =$	$2 \times 4 =$
$5 \times 1 =$	$6 \times 8 =$	$7 \times 9 =$	$8 \times 10 =$
$0 \times 10 =$	$1 \times 11 =$	$7 \times 3 =$	$11 \times 9 =$
$6 \times 4 =$	$7 \times 11 =$	$3 \times 7 =$	$4 \times 5 =$
$9 \times 5 =$	$10 \times 6 =$	$12 \times 10 =$	$9 \times 12 =$

(Be sure that you can easily do problems like these: arithmetic with whole numbers, decimals, fractions; arithmetic with signed numbers)

$\frac{0}{7.2} =$	$-\frac{(6)(-2)}{-3} =$	$-3-(-2) =$
$1,000 \times 3.47 =$	$\frac{248.36}{100} =$	$\frac{1}{3}-\frac{1}{5} =$
$\frac{1}{3} \cdot \frac{1}{5} =$	$\frac{1}{3} \div \frac{1}{5} =$	$126 \times 24 =$

SAMPLE PREREQUISITE PROBLEMS: ALGEBRA II

Problems 1–9 should be done WITHOUT A CALCULATOR.

1. For each expression given below, rename the expression as requested. If the requested name is not possible, so state. A few samples are done for you.

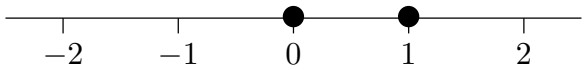
EXPRESSION	RENAME IN THIS FORM	PUT YOUR ANSWER HERE
(sample) 12	a sum of even integers	$2+10$ or $4+8$ etc.
(sample) 12	xy , where x and y are integers, with $x<0$	$(-3)(-4)$ or $(-2)(-6)$ etc.
(sample) 12	2^x , where $x \in \{0,1,2,3,\dots\}$	not possible
$\dfrac{1}{\sqrt{2}}$	a fraction with no radical in the denominator	
7	a quotient of integers, where the numerator is greater than 10	
23,070,000	in scientific notation	
7	$x-y$, where x and y are NOT integers	
7	$\frac{1}{2}\cdot x$	
$(x-2)(x+3)$	as a sum (i.e., multiply out)	
x^2-y^2	as a product (i.e., factor)	
$\frac{1}{2}$	$\frac{3}{x}$	
$\dfrac{1}{x}-\dfrac{2}{3+x}$	as a single fraction	
0.25	as a percent	
$\dfrac{x^4x^{-1}}{(x^2)^3x}$	x^k	
300 ft/sec	x mph (there are $5,280$ feet in one mile)	
7,036	$x_1\cdot 1000+x_2\cdot 100+x_3\cdot 10+x_4\cdot 1$	
7,036	$x\cdot 1000$	
7,036	$x\cdot 10$	
$8^{-2/3}$	as a simple fraction	

2. Solve each equation/inequality in one variable. Write a list of equivalent sentences, ending with one that can be solved by inspection. Get EXACT answer(s), not decimal approximations. Graph each solution set on a number line. A sample is done for you.

(sample) $x^2 = x$

Solution:

$$x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0 \text{ or } x - 1 = 0$$
$$x = 0 \text{ or } x = 1$$



(a) $3x(1-5x)(x^2-16) = 0$

(b) $\frac{1}{2}x - 7 = 3x + \frac{x}{5}$

(c) $|x-3| > 1$

(d) $1 - 2x < 3$

(e) $x^2 = x + 2$

(f) $1 < x$ or $x \leq -1$

(g) $0.4(x-1) = 2x - 3$

(h) $x^2 = 3$

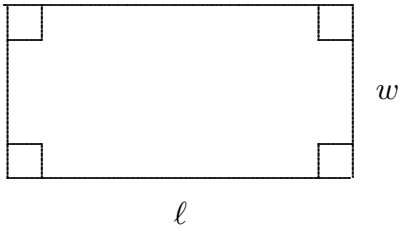
(i) $-1 \leq x < 3$

(j) $x > -1$ and $x \leq 3$

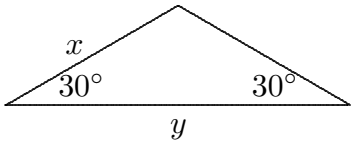
3. Write the equation of the circle with center $(1, -2)$ and radius 5.

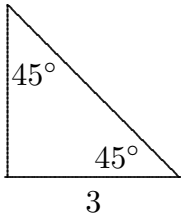
4. Find the requested measurement(s) of each geometric figure.

(a) PERIMETER and AREA:

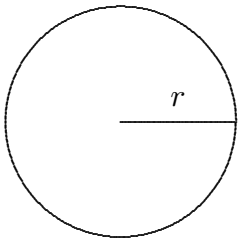


(b) PERIMETER and AREA:

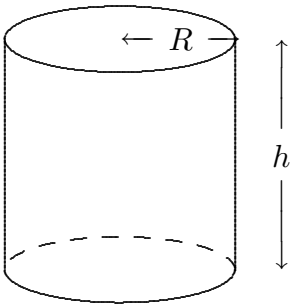




(c) CIRCUMFERENCE and AREA:



(d) VOLUME:



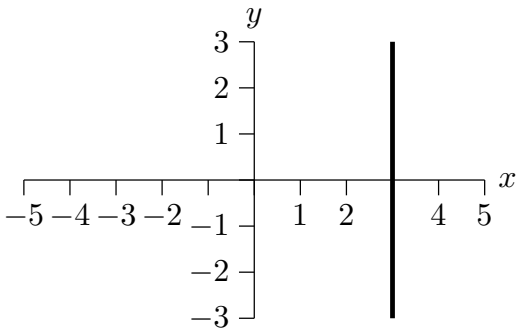
Which of the units below is a unit of length? Of area? Of volume?

cubic feet $\quad$ cm^2 $\quad$ meter

5. Graph each of the following equations/inequalities, where each sentence is viewed as a sentence in two variables. (That is, $x = 3$ should be viewed as $x + 0y = 3$.) A sample is done for you.

(sample) $x = 3$

Solution:



(a) $x > 3$

(b) $2y - 3 = 0$

(c) $x = 3$ and $y = 2$

(d) $x = 3$ or $y = 2$

(e) $y = 2x - 1$

(f) $y = \sqrt{x}$

(g) $|x| = 2$

(h) $y \leq 2$

(i) $\frac{y-2}{3} = 2x - 1$

(j) $(x + 1)^2 + (y - 3)^2 = 25$

6. Write an expression (using the variable x) to represent each sequence of operations.

(a) take a number, multiply by 2, then subtract 3

(b) take a number, subtract 3, then multiply by 2

(c) take a number, multiply it by 2, cube the result, add 1, then divide by the original number

Write the sequence of operations that is being described by each expression.

(d) $3x - 1$

(e) $2(x + 1)^3 - 5$

(f) $\dfrac{x - 3}{7} - 1$

7. Let $f(x) = x^2 - 2x + 1$. Evaluate each of the following expressions.

   (a) $f(0)$

   (b) $f(1) - 2$

8. Find the domain of the function $g(x) = \frac{1}{\sqrt{x-3}}$. Report your answer using interval notation.

9. Write the equation of the line, in $y = mx+b$ form, that satisfies the given conditions.

   (a) slope 3, passing through the point $(2, -1)$

   (b) the horizontal line that crosses the y-axis at 2

   (c) the line that is perpendicular to $x - 3y = 5$ and passes through the point $(0, 3)$

10. (Your calculator is needed for parts of this question.)

    (a) What is the domain of the function $f(x) = \dfrac{1 - 3x}{x - 2}$?

    (b) Use your graphing calculator to graph the function f in the window $-1 < x < 3$ and $-15 < y < 10$.

    (c) Find the x-intercept of the graph.

    (d) Use your calculator to estimate a value for x for which $f(x) = 5$. (Zoom, as necessary, to get f(x) within 0.01 of 5.)

11. Estimate (where necessary) each of the following numbers on your calculator. For full credit, each answer must be correct to five decimal places.

    (a) $\dfrac{1 + \sqrt{2}}{\sqrt[3]{5} - 7}$

    (b) $3x^2 - 5x + 1$, where $x = -1.8$

    (c) $|1 - 2x|$, where $x = \sqrt{3}$

    (d) $(2.03 \times 10^{-9})(-4.1 \times 10^{7})$

SOLUTIONS

Multiplication Tables:
12, 6, 36, 10
64, 27, 70, 8
5, 48, 63, 80
0, 11, 21, 99
24, 77, 21, 20
45, 60, 120, 108

$0\,,\ -4\,,\ -1$ $\qquad$ $3{,}470\,,\ 2.4836\,,\ \frac{2}{15}$ $\qquad$ $\frac{1}{15}\,,\ \frac{5}{3}\,,\ 3{,}024$

1. There are many possible correct answers for this problem.

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$7 = 7 \cdot \frac{11}{11} = \frac{77}{11}$

$23,070,000 = 2.307 \times 10^7$

$7 = 7 + .2 - .2 = 7.2 - .2$

$7 = 7 \cdot \frac{1}{2} \cdot 2 = \frac{1}{2} \cdot 14$

$(x-2)(x+3) = x^2 + 3x - 2x - 6 = x^2 + x - 6$

$x^2 - y^2 = (x-y)(x+y)$

$\frac{1}{2} = \frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$

$\frac{1}{x} - \frac{2}{3+x} = \frac{1}{x} \cdot \frac{3+x}{3+x} - \frac{2}{3+x} \cdot \frac{x}{x} = \frac{3+x-2x}{x(3+x)} = \frac{3-x}{x(3+x)}$

$0.25 = \frac{25}{100} = 25 \cdot \frac{1}{100} = 25\%$

$\frac{x^4 x^{-1}}{(x^2)^3 x} = \frac{x^3}{x^7} = x^{3-7} = x^{-4}$

$300\frac{\text{ft}}{\text{sec}} = 300\frac{\text{ft}}{\text{sec}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 204.5\frac{\text{miles}}{\text{hr}}$

$7,036 = 7 \cdot 1000 + 0 \cdot 100 + 3 \cdot 10 + 6 \cdot 1$

$7,036 = 7.036 \cdot 1000$

$7,036 = 703.6 \cdot 10$

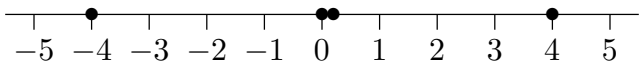
$8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{(8^{1/3})^2} = \frac{1}{2^2} = \frac{1}{4}$

2.

$$\text{(a)} \quad 3x(1-5x)(x^2-16) = 0$$

$$x = 0 \quad \text{or} \quad 1 - 5x = 0 \quad \text{or} \quad x^2 - 16 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{1}{5} \quad \text{or} \quad x = \pm 4$$

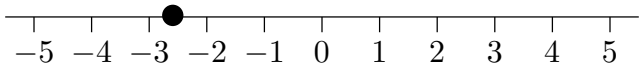


(b) $\frac{1}{2}x - 7 = 3x + \frac{x}{5}$

$5x - 70 = 30x + 2x$ (clear fractions; multiply by 10)

$-70 = 27x$

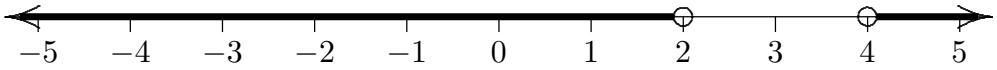
$x = \frac{-70}{27}$



(c) $|x - 3| > 1$

solve by inspection; want all #s whose distance from 3 is greater than 1

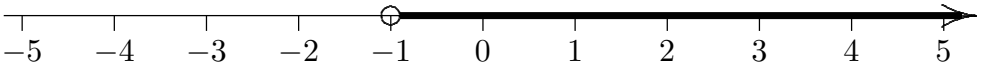
$x < 2$ or $x > 4$



(d) $1 - 2x < 3$

$-2x < 2$

$x > -1$

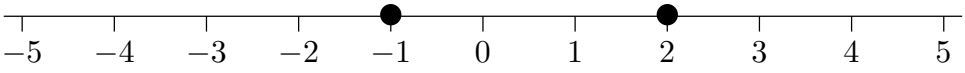


(e) $x^2 = x + 2$

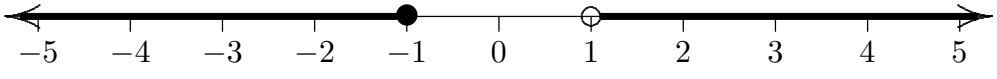
$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$



(f) $1 < x$ or $x \leq -1$



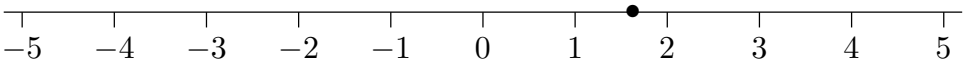
(g) $0.4(x-1) = 2x - 3$

$4(x-1) = 20x - 30$ (simplify numbers; multiply by 10)

$4x - 4 = 20x - 30$

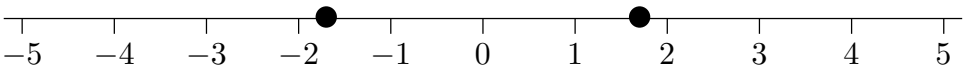
$26 = 16x$

$x = \dfrac{26}{16} = \dfrac{13}{8}$

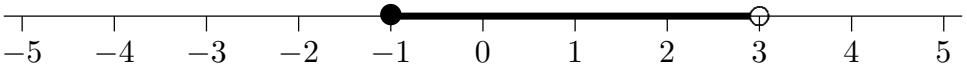


(h) $x^2 = 3$

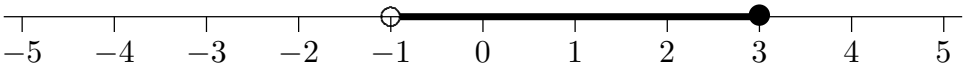
$x = \pm\sqrt{3}$



(i) $-1 \le x < 3$



(j) $x > -1$ and $x \le 3$



3.

$(x-1)^2 + (y-(2))^2 = 5^2$

$(x-1)^2 + (y+2)^2 = 25$

4. (a) PERIMETER $= 2\ell + 2w$, AREA $= \ell w$

(b) First triangle: PERIMETER $= 2x + y$, AREA $= \frac{1}{2}(y)(\frac{x}{2}) = \frac{1}{4}xy$

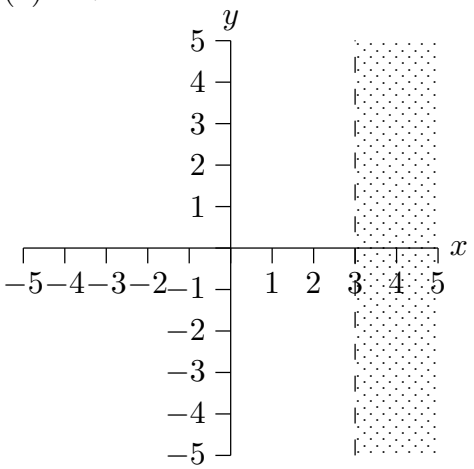
Second triangle: PERIMETER $= 3 + 3 + 3\sqrt{2} = 6 + 3\sqrt{2}$, AREA $= \frac{1}{2}(3)(3) = \frac{9}{2}$.

(c) CIRCUMFERENCE $= 2\pi r$, AREA $= \pi r^2$

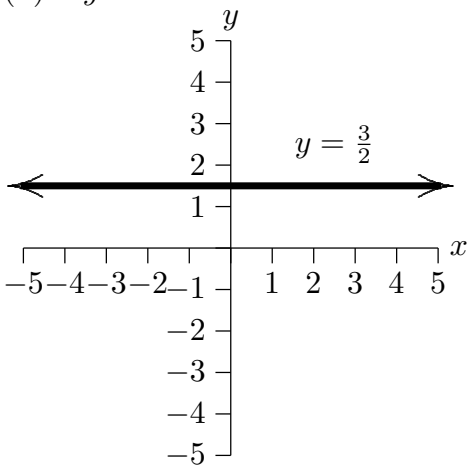
(d) VOLUME = (area of base)(height) $= \pi R^2 h$

Meter is a unit of length; cm^2 is a unit of area; cubic feet is a unit of volume.

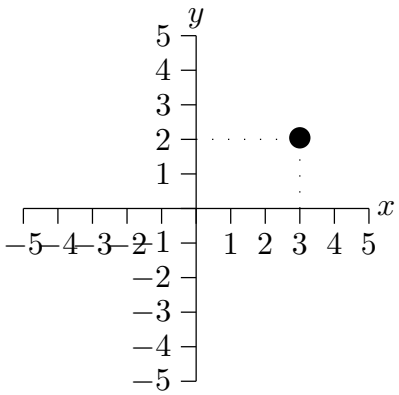
5. (a) $x > 3$



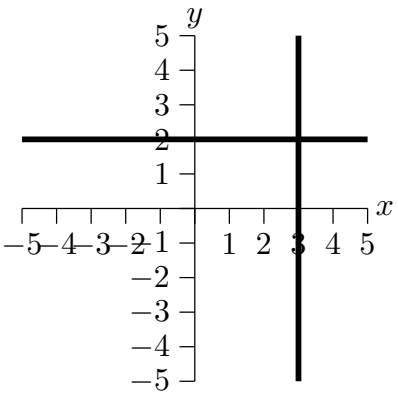
(b) $2y - 3 = 0$



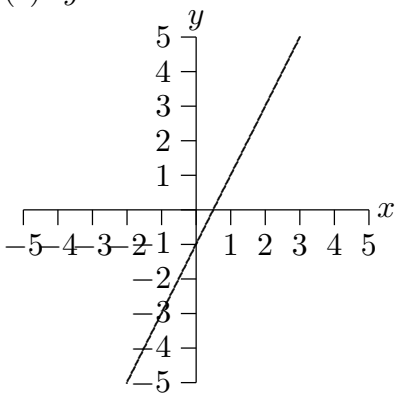
(c) $x = 3$ and $y = 2$



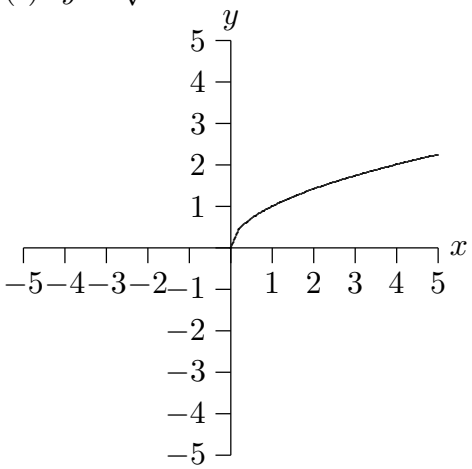
(d) $x = 3$ or $y = 2$



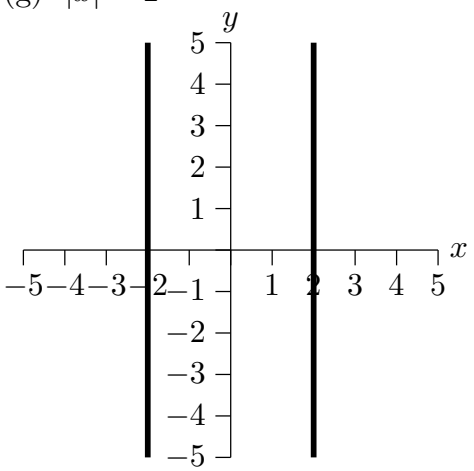
(e) $y = 2x - 1$



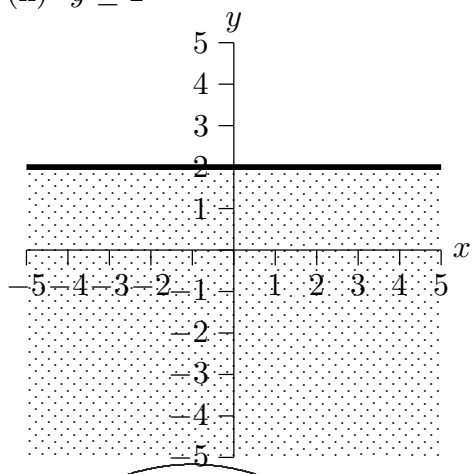
(f) $y = \sqrt{x}$



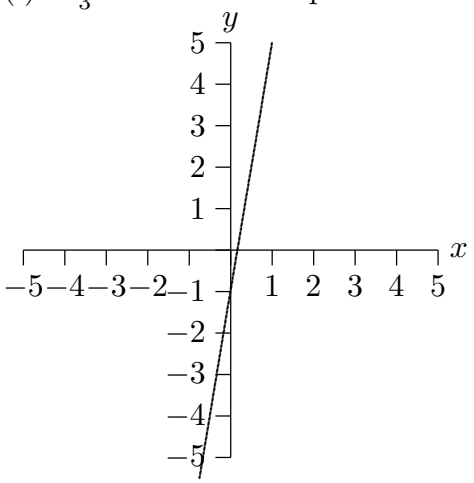
(g) $|x| = 2$



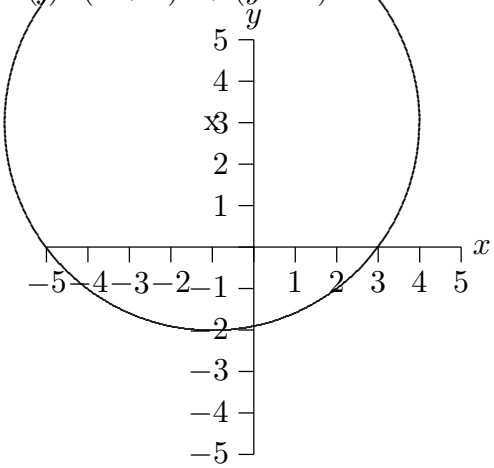
(h) $y \leq 2$



(i) $\frac{y-2}{3} = 2x - 1$ is equivalent to $y = 6x - 1$



(j) $(x+1)^2 + (y-3)^2 = 25$



6. (a) $2x - 3$

(b) $2(x - 3)$

(c) $\frac{(2x)^3+1}{x}$

(d) take a number, multiply by 3, then subtract 1

(e) take a number, add 1, cube the result, multiply by 2, then subtract 5

(f) take a number, subtract 3, divide by 7, then subtract 1

7. (a) $f(0) = 0^2 - 2(0) + 1 = 1$

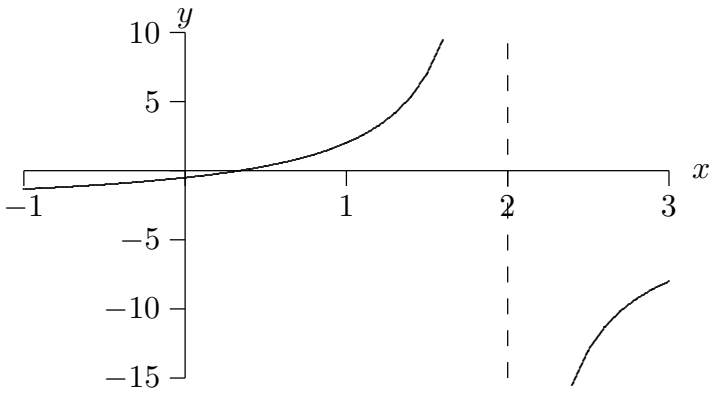
(b) $f(1) - 2 = \left(1^2 - 2 \cdot 1 + 1\right) - 2 = 0 - 2 = -2$

8. The function g is defined whenever $x - 3 > 0$, that is, whenever $x > 3$.
The domain of g is the interval $(3, \infty)$.

9. (a) $y = 3x - 7$
(b) $y = 2$
(c) The line $x - 3y = 5$ has slope $\frac{1}{3}$; a perpendicular line will have slope -3.
The line with slope -3 passing through $(0, 3)$ has equation $y = -3x + 3$.

10. (a) The domain of f is the set of all real numbers except 2.

(b)



(c) The graph crosses the x-axis at $\frac{1}{3}$. (Set $1 - 3x = 0$. Be sure you can get this *exact* answer, not just $x \approx 0.333333$.)
(d) When $x = 1.375$ (exactly), then $f(x) = 5$. (You could check this, if desired, by solving the equation $5 = \frac{1-3x}{x-2}$.)

11. (a) $\frac{1+\sqrt{2}}{\sqrt[3]{5}-7} \approx -0.45637$
(b) 19.72 (this is exact)
(c) $|1 - 2\sqrt{3}| \approx 2.46410$
(d) $(2.03 \times 10^{-9})(-4.1 \times 10^{7}) = -0.08323$ (this is exact)